VOLUME 78

SEPARATE No. 116

PROCEEDINGS

AMERICAN SOCIETY OF CIVIL ENGINEERS

FEBRUARY, 1952

AMERICAN SOCIETY OF CIVIL ENGINEERS FOUNDED 1852

GRAPHICAL SOLUTION OF HYDRAULIC PROBLEMS

By Kenneth E. Sorensen, Jun. M. ASCE

ENGINEERING MECHANICS DIVISION

Headquarters of the Society
33 W. 39th St.
New York 18, N.Y.

PRICE $0.50 PER COPY

GUIDEPOST FOR TECHNICAL READERS

"Proceedings-Separates" of value or significance to readers in various fields are here listed, for convenience, in terms of the Society's Technical Divisions. Where there seems to be an overlapping of interest between Divisions, the same Separate number may appear under more than one item.

Technical Division	*Proceedings-Separate Number*
Air Transport	42, 43, 48, 52, 60, 93, 94, 95, 100, 103, 104, 108 (Discussion: D-XXVIII, D-7, D-16, D-18, D-23, D-43)
City Planning	58, 60, 62, 64, 93, 94. 99, 101, 104, 105, 115 (Discussion: D-16, D-23, D-43, D-60, D-62)
Construction	43, 50, 55, 71, 92, 94, 103, 108, 109, 113, 117 (Discussion: D-3, D-8, D-17, D-23, D-36, D-40)
Engineering Economics	46, 47, 62, 64, 65, 68, 69, 95, 100, 104 (Discussion: D-2, D-19, D-27, D-30, D-36, D-57)
Engineering Mechanics	41, 49, 51, 54, 56, 59, 61, 88, 89, 96, 116 (Discussion: D-5, D-XXIII, D-XXV, D-18, D-24, D-33, D-34 D-49, D-54, D-61)
Highway	43, 44, 48, 58, 70, 100, 105, 108, 113 (Discussion: D-XXVIII, D-7, D-13, D-16, D-23, D-60)
Hydraulics	50, 55, 56, 57, 70, 71, 78, 79, 80, 83, 86, 92, 96, 106, 107, 110, 111, 112, 113, 116 (Discussion: D-XXVII, D-9, D-11, D-19, D-28, D-29, D-56, D-70)
Irrigation	46, 47, 48, 55, 56, 57, 67, 70, 71, 87, 88, 90, 91, 96, 97, 98, 99, 102, 106, 109, 110, 111, 112, 114, 117, 118 (Discussion: D-XXIII, D-3, D-7, D-11, D-17, D-19, D-25-K, D-29, D-30, D-38, D-40, D-44, D-47, D-57, D 70)
Power	48, 55, 56, 69, 71, 88, 96, 103, 106, 109, 110, 117, 118 (Discussion: D-XXIII, D-2, D-3, D-7, D-11, D-17, D-19, D-25-K, D-30, D-38, D-40, D-44, D-70)
Sanitary Engineering	55, 56, 87, 91, 96, 106, 111, 118 (Discussion: D-10, D-29, D-37, D-56, D-60, D-70)
Soil Mechanics and Foundations	43, 44, 48, 94, 102, 103, 106, 108, 109, 115 (Discussion: D-4, D-XXVIII, D-7, D-43, D-44, D-56)
Structural	42, 49, 51, 53, 54, 59, 61, 66, 89, 100, 103, 109, 113, 116, 117 (Discussion: D-3, D-5, D-8, D-13, D-16, D-17, D-21, D-23, D-24, D-25-K, D-32, D-33, D-34, D-37, D-39, D-42, D-49, D-51, D-54, D-59, D-61)
Surveying and Mapping	50, 52, 55, 60, 63, 65, 68 (Discussion: D-60)
Waterways	41, 44, 45, 50, 56, 57, 70, 71, 96, 107, 112, 113, 115 (Discussion: D-8, D-9, D-19, D-27, D-28, D-56, D-70)

A constant effort is made to supply technical material to Society members, over the entire range of possible interest. Insofar as your specialty may be covered inadequately in the foregoing list, this fact is a gage of the need for your help toward improvement. Those who are planning papers for submission to "Proceedings-Separates" will expedite Division and Committee action measurably by first studying the ASCE "Guide for Development of Proceedings-Separates" as to style, content, and format. For a copy of this Manual, address the Manager, Technical Publications, ASCE, 33 W. 39th Street, New York 18, N. Y.

The Society is not responsible for any statement made or opinion expressed in its publications

Published at Prince and Lemon Streets, Lancaster, Pa., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-ninth Street, New York 18, N. Y. Reprints from this publication may be made on condition that the full title of paper, name of author, page reference, and date of publication by the Society are given.

Proceedings—Vol. 78 February, 1952 Separate No. 116

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

GRAPHICAL SOLUTION OF HYDRAULIC PROBLEMS

By Kenneth E. Sorensen,[1] Jun. M. ASCE

Synopsis

The graphical solution of first order differential equations provides a simple and flexible means of solving several common and important problems in the field of hydraulics.

The method of graphical solution is developed and applied to problems involving reservoir flood routing, water power studies, the tracing of water surface curves in nonuniform channel flow, and the determination of water surface variation in surge tanks. Several examples are presented and solved.

The method of extending the graphical solution to second order differential equations is also explained.

Introduction

Several common and important problems in the field of hydraulics require the solution of a differential equation of the first order in the form

$$d\frac{f_1(y)}{dx} + f_2(y) = f_3(x) \dots\dots\dots\dots\dots\dots\dots (1)$$

in which x is the independent variable and y is the dependent variable. A graphical solution of this equation is possible if the functions $f_1(y)$, $f_2(y)$, and $f_3(x)$, and the values of x_1 and y_1 at some one point are known. Eq. 1 may be approximated as

$$\Delta f_1(y) = f_1(y_2) - f_1(y_1) = [f_3(x) - f_2(y)]\,\Delta x \dots\dots\dots\dots (2)$$

Furthermore, if the assumption is made that

$$f_2(y) = \tfrac{1}{2}\,[f_2(y_2) + f_2(y_1)] \dots\dots\dots\dots\dots\dots (3)$$

Note.—Written comments are invited for publication; the last discussion should be submitted by August 1, 1952.

[1] Structural Engr., Harza Eng. Co., Chicago, Ill.

and that $f_3(\bar{x})$ is the mean value of $f_3(x)$ between points (x_1, y_1) and (x_2, y_2) then

$$f_1(y_2) - f_1(y_1) = [f_3(\bar{x}) - f_2(y_1)]\frac{\Delta x}{2} + [f_3(\bar{x}) - f_2(y_2)]\frac{\Delta x}{2} \ldots\ldots (4)$$

Fig. 1 demonstrates a graphical method for solving Eq. 4. Curve 1 is plotted with $f_1(y)$ as ordinates and $f_2(y)$ as abscissas. Line 2 is then drawn vertically at a distance from the origin equal to $f_3(\bar{x})$. If, from point y_1 on

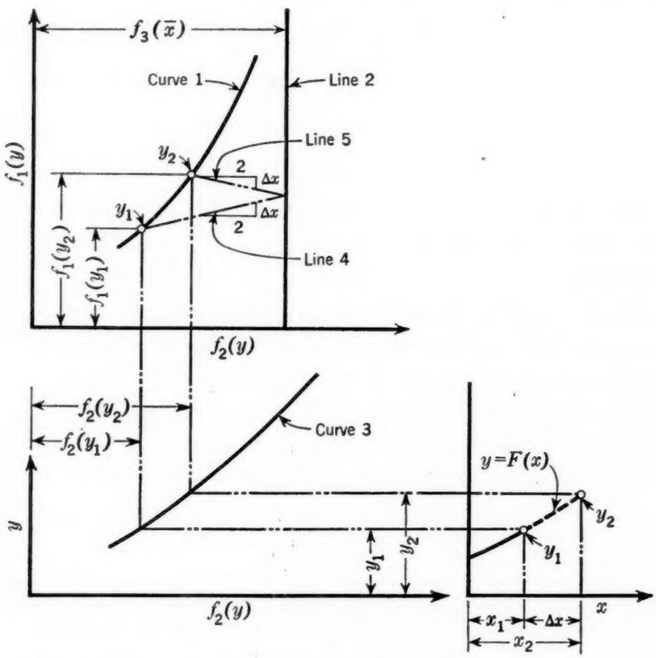


FIG. 1.—GRAPHICAL SOLUTION OF A FIRST ORDER DIFFERENTIAL EQUATION

curve 1, line 4 is drawn at slope $\frac{\Delta x}{2}$ to intersect line 2, and if from that intersection line 5 is drawn at slope $-\frac{\Delta x}{2}$ to intersect curve 1 at y_2, then

$$f_1(y_2) - f_1(y_1) = [f_3(\bar{x}) - f_2(y_1)]\frac{\Delta x}{2} + [f_3(\bar{x}) - f_2(y_2)]\frac{\Delta x}{2} \ldots\ldots (5)$$

which is identical to Eq. 4. To permit a plot of $y = f(x)$, curve 3 is drawn with y as ordinates and $f_2(y)$ as abscissas. By projecting from curve 1 through curve 3, each new value of y_2 obtained may be plotted opposite the corresponding value of x. Once y_2 has been established, it then serves as y_1 for the next interval Δx, and graphical process is repeated.

Applications to Hydraulics

A number of problems encountered in the field of hydraulics may be conveniently solved by the foregoing method. Examples of such problems are flood routing for reservoirs, water power studies, nonuniform flow in channels, and surge tank studies.

Reservoir Flood Routing.—The equation for reservoir flood routing is

$$\frac{dC}{dT} = I - Q \qquad (6)$$

in which C = reservoir volume $= f_1(h)$ (h equals the reservoir elevation); I = inflow $= f_3(T)$ (T equals the time); and Q = outflow $= f_2(h)$; so that

$$\frac{df_1(h)}{dT} + f_2(h) = f_3(T) \qquad (7)$$

This equation is in the same form as Eq. 1.

Fig. 2(a) shows the graphical solution of Eqs. 6 and 7. Curve 1 is plotted with coordinates Q and C and curve 3 is plotted with coordinates h and C (reservoir volume curve). Both h and Q can be plotted as functions of time as the graphical solution progresses. As $f_3(t) = I$, it is convenient to plot the inflow hydrograph to the same scale as Q. Then line 2 can be drawn by projecting horizontally from the average value of I for each period, ΔT.

If wedge storage is to be considered and its quantity is known, a family of curves may be drawn for curve 1, each labeled with the appropriate value of I. Then, as shown in Fig. 2(b), the curve corresponding to I_1 is used for the initial point (h_1), and the curve corresponding to I_2 is used for the final point (h_2). However, curve 1, corresponding to zero wedge storage, must be used for projecting to curve 3 in obtaining the plot of h against time.

Water Power Studies.—Closely allied to reservoir flood routing is the water power study involving determination of reservoir drawdown required to maintain constant power output of a plant under varying conditions of reservoir inflow. The equation for power is

$$P_h = Q f'_2(h) \qquad (8a)$$

or

$$Q = f_2(h, P_h) \qquad (8b)$$

in which Q is the discharge through the turbines, P_h is the horse power, and $f_2(h, P_h)$ is a function of reservoir elevation that includes the effect of tailwater rise and the hydraulic and mechanical efficiency.

If $f_2(h, P_h)$ is substituted for $f_2(h)$ in Eq. 7, then the method described for Fig. 2 will permit a plot of reservoir elevation against time for any specific power output.

Nonuniform Flow in Channels.—Bernoulli's theorem for flow in channels (Fig. 3(a)) may be expressed

$$\frac{dE}{dL} + S = 0 \qquad (9)$$

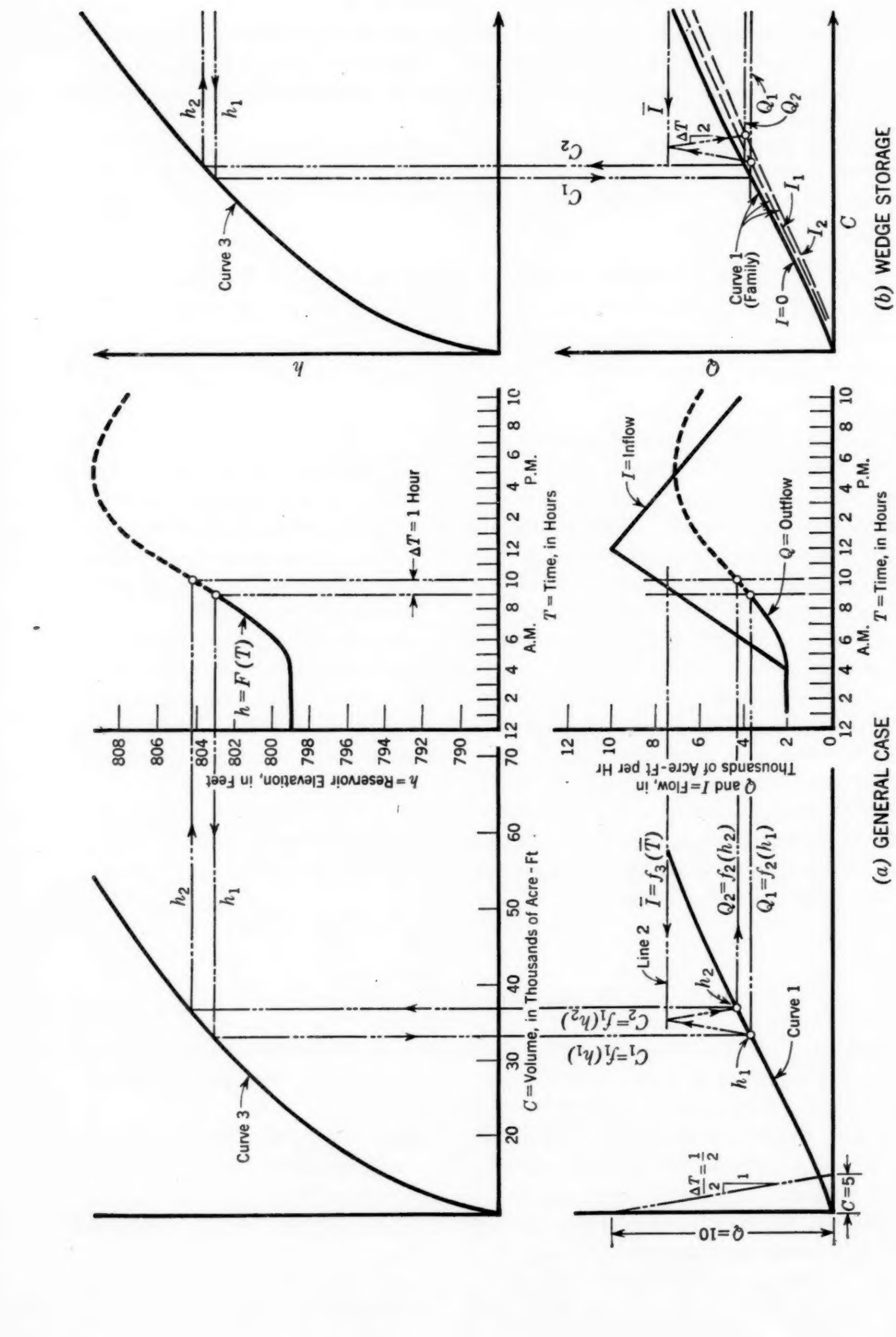


FIG. 2.—GRAPHICAL SOLUTION OF RESERVOIR FLOOD ROUTING

in which E = total energy $= D + \frac{V^2}{2\,g} - S_o L$; D is the depth of water; V is the velocity; S_o = bottom slope $= f_3(L)$; L is the length of reach measured along channel bottom; and S = friction slope $= f_2(D)$. If Q is constant, $D + \frac{V^2}{2\,g} = f_1(D)$, and Eq. 9 then takes the form

$$\frac{df_1(D)}{dL} + f_2(D) = f_3(L) \qquad (10)$$

In the case of certain symmetrical artificial channels, such as those shown in Fig. 3(b), it is desirable to alter Eq. 10 somewhat. In accordance with gener-

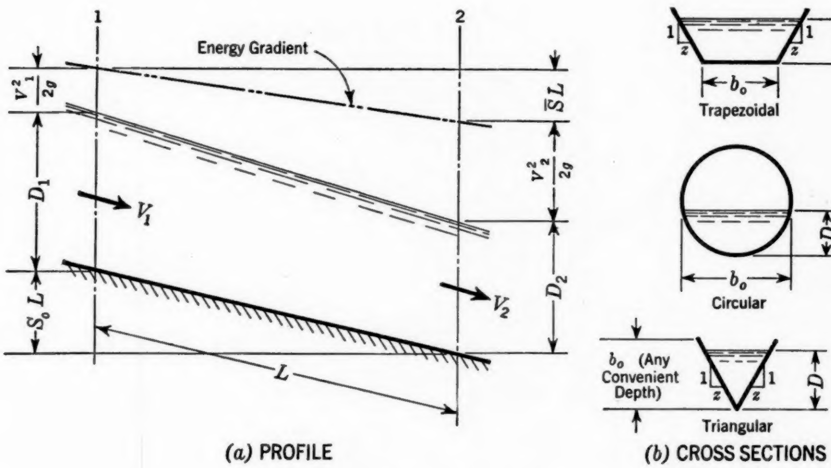


FIG. 3.—FLOW IN CHANNELS

ally accepted hydraulic practice

$$f_1(D) = D + \frac{V^2}{2\,g} = D + \frac{Q^2}{2\,g\,A^2} \qquad (11)$$

in which A = the area $= a\,b^2_o$ (b_o is as defined in Fig. 3(b) and

$$f_2(D) = S = \frac{Q^2\,n^2}{2.2082\,A^2\,R^{4/3}} \qquad (12)$$

in which n = the coefficient of roughness, and R = the hydraulic radius $= r\,b_o$. Letting

$$a = F_1\left(\frac{D}{b_o}\right) \qquad (13)$$

and

$$r = F_2\left(\frac{D}{b_o}\right) \qquad (14)$$

Then

$$f_1(D) = b_o \frac{D}{b_o} + \frac{Q^2}{2\,g\,a^2\,b^4_o} \quad \text{(15a)}$$

and

$$f_2(D) = \frac{Q^2\,n^2}{2.2082\,a^2\,r^{4/3}\,(b_o)^{16/3}} \quad \text{(15b)}$$

Further, if

$$\rho = \frac{D}{b_o} \quad \text{(16a)}$$

$$\phi(\rho) = \frac{1}{2\,g\,a^2} \quad \text{(16b)}$$

$$\psi(\rho) = \frac{1}{2.2082\,a^2\,r^{4/3}} \quad \text{(16c)}$$

$$M = \frac{b^5_o}{Q^2} \quad \text{(16d)}$$

$$N = \frac{S_o\,b^4_o}{Q^2} \quad \text{(16e)}$$

and

$$P = \frac{n^2}{b^{4/3}_o} \quad \text{(16f)}$$

then

$$f_1(D) = \frac{b_o}{M}\left[\rho\,M + \phi(\rho)\right] \quad \text{(17a)}$$

$$f_2(D) = \frac{b_o}{M}\left[P\,\psi(\rho)\right] \quad \text{(17b)}$$

and

$$f_3(L) = S_o = \frac{b_o}{M}\,N \quad \text{(17c)}$$

and Eqs. 9 and 10 can be written

$$\frac{d\left[\rho\,M + \phi(\rho)\right.}{P\,dL} + \psi(\rho) = \frac{N}{P} \quad \text{(18)}$$

Fig. 4 shows a graphical solution of Eq. 18. Curve 1 is plotted with ordinates of $\rho\,M + \phi(\rho)$ and abscissas of $\psi(\rho)$. Line 2 is drawn at a distance $\frac{N}{P}$ from the origin. Curve 3 is drawn with ordinates of ρ (or D) and with abscissas of $\psi(\rho)$. Lines 4 and 5 are drawn at slopes $\frac{P\,\Delta L}{2}$ in the process of the graphical solution.

Certain characteristics of Eq. 18 and Fig. 4 are worthy of mention:

1. The functions $\phi(\rho)$ and $\psi(\rho)$ are functions of channel shape only;
2. Curve 1 is dependent on the factors of channel shape, Q and b_o only;
3. Distance $\frac{N}{P}$ is dependent on the values of Q, b_o, S_o, and n only;

4. The slope $\dfrac{P\,\Delta L}{2}$ is dependent on the factors b_o, n, and ΔL only;

5. The low point of curve 1 occurs at the value of $\rho_c = \dfrac{D_c}{b_o}$ in which D_c is the critical depth of the channel for flow Q;

6. The intersection of line 2 with curve 1 coincides with the value of $\rho_o = \dfrac{D_o}{b_o}$, in which D_o is the depth of channel at which uniform flow occurs.

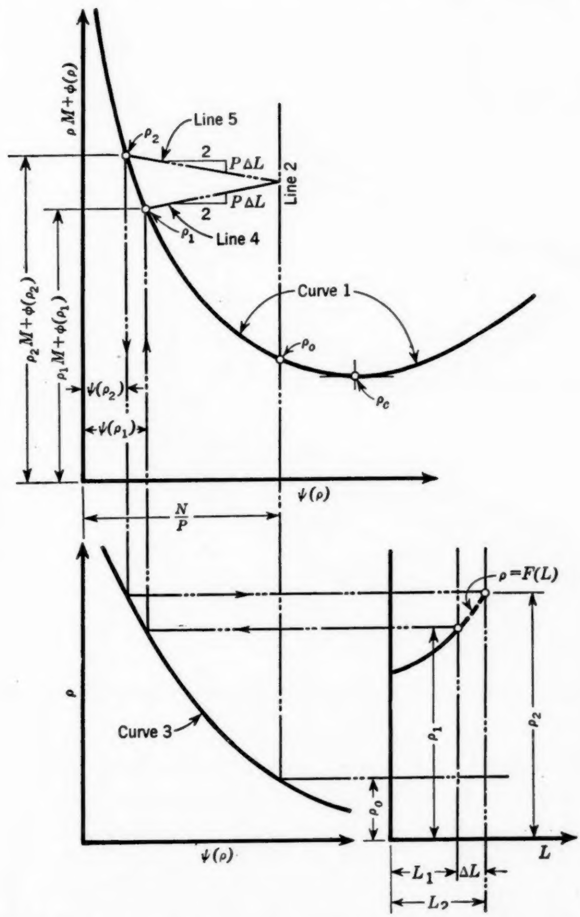


FIG. 4.—GRAPHICAL SOLUTION OF NONUNIFORM FLOW IN CHANNELS

It should be noted that the solution described is the reverse of the usual methods, in that depths are determined at given increments of length, rather than the lengths being determined between given increments of depth. No trial and error is involved, not even for cases in which the values of S_o and n may vary at points along the channel. If, at some point in the water surface

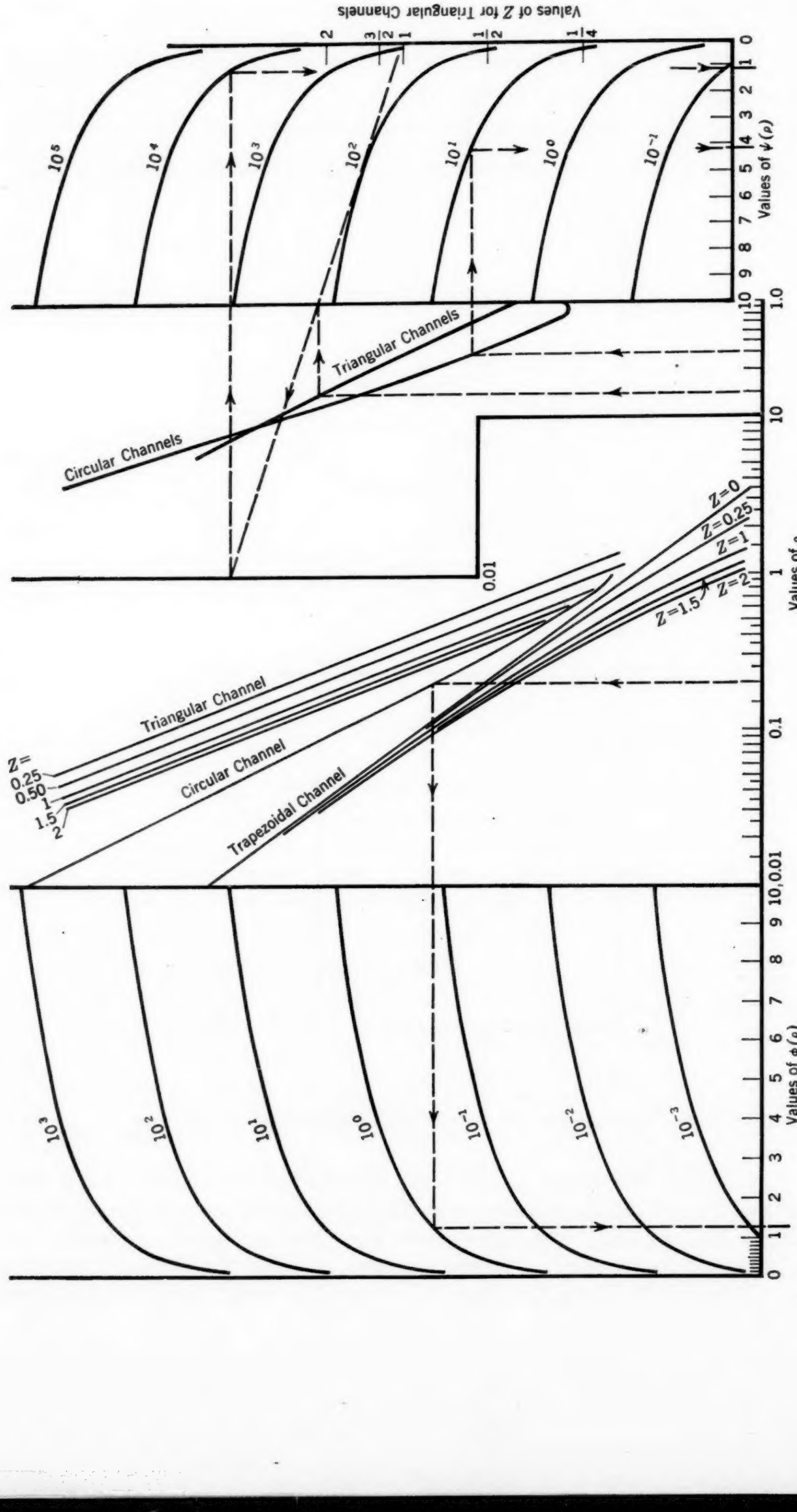


Fig. 5.—Chart for Determining Values of $\phi(\rho)$ and $\psi(\rho)$

plot, it is desired to change the increment of length (ΔL) being used, only the slopes of lines 4 and 5 are affected. The smaller the value of ΔL is chosen, the more accurate the solution becomes, particularly in regions in which the curvature of curve 1 is sharp.

Values of $\phi(\rho)$ for the channels shown in Fig. 3(*b*) can be obtained from Fig. 5. The values of $\psi(\rho)$ for circular and triangular channels can also be obtained from Fig. 5. Values of $\psi(\rho)$ for rectangular and trapezoidal channels can be obtained from the "Handbook of Hydraulics" by Horace W. King,[2] M. ASCE, as $\psi(\rho)$ of Eq. 18, is identical with $\left(\frac{1}{K'}\right)^2$ as used by Mr. King.

Fig. 6 shows a sample problem for the solution of a backwater curve in a rectangular channel.

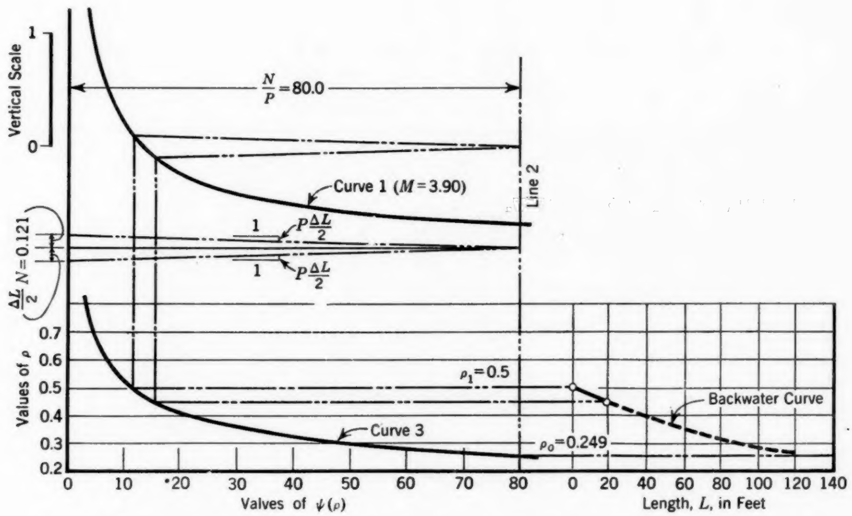


FIG. 6.—GRAPHICAL DETERMINATION OF A BACKWATER CURVE IN A RECTANGULAR CHANNEL

The desired ΔL interval for this problem is 20 ft. The other factors involved are $b_o = 0.654$ ft; $Q = 0.1732$ cu ft per sec; $n = 0.00935$; and $S_o = 0.002$. By use of the equations previously derived it can be determined that $M = 3.90$, $\frac{\Delta L}{2} N = 0.121$, and $\frac{N}{P} = 80.0$. The determination of the backwater curve then follows the procedure outlined in Fig. 6.

In the case of natural channels or artificial channels of more complicated section, it is more convenient to express the functions in terms of D rather than ρ. If $b_o = 1$, then Eq. 18 becomes

$$\frac{d\,[D\,M' + \Phi(D)]}{d\,\frac{P'}{L}} + \Psi(D) = \frac{N'}{P'} \qquad (19)$$

[2] "Handbook of Hydraulics for the Solution of Hydraulic Problems," by Horace W. King, McGraw-Hill Book Co., Inc., New York, N. Y., 3d Ed., 1939, pp. 341–356, Table 114.

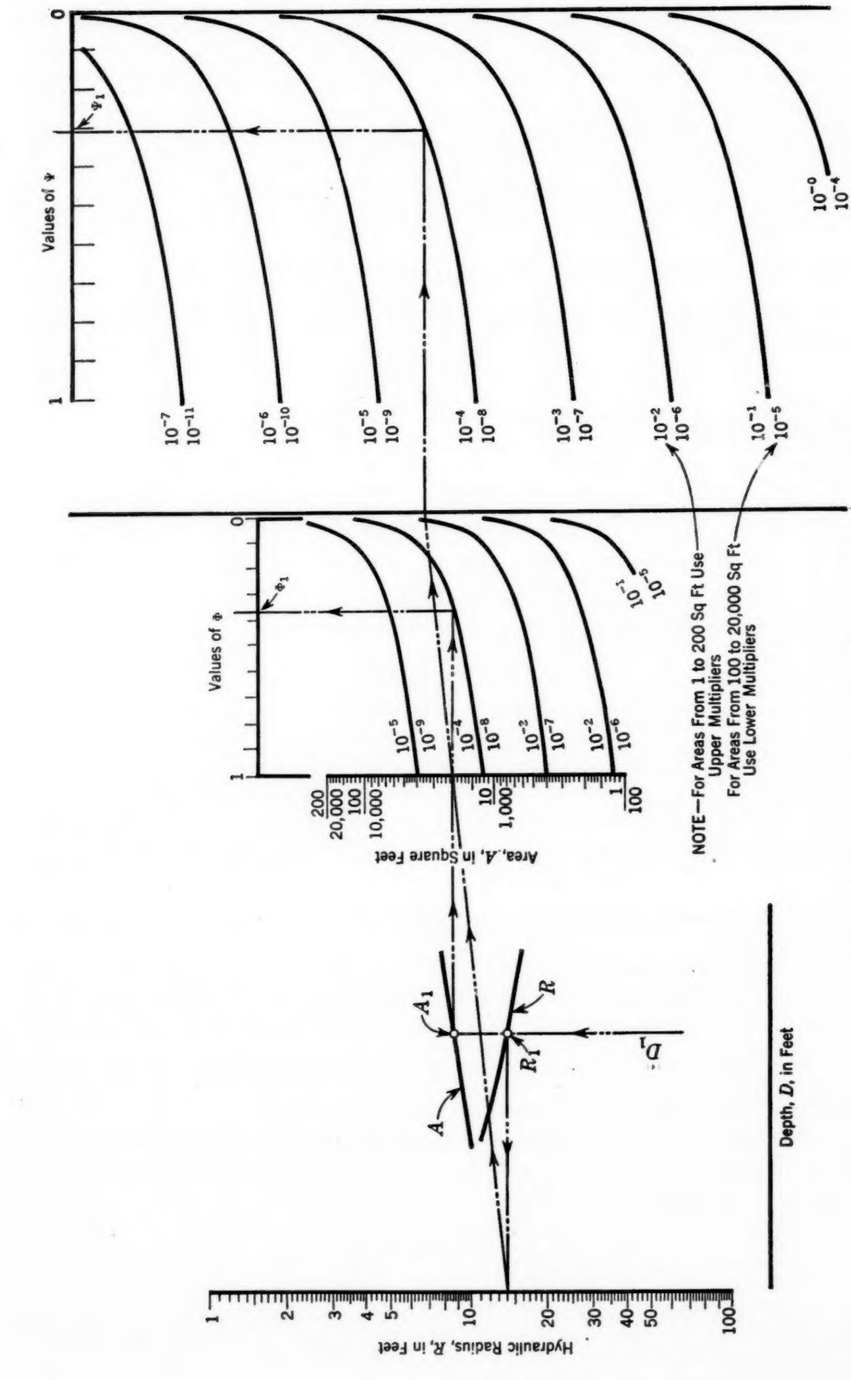


FIG. 7.—CHART FOR DETERMINING VALUES OF $\Phi(D)$ AND $\Psi(D)$

in which

$$\Phi(D) = \frac{1}{2\,g\,A^2} \quad \text{(20a)}$$

$$\Psi(D) = \frac{1}{2.2082\,A^2\,R^{4/3}} \quad \text{(20b)}$$

$$M' = \frac{1}{Q^2} \quad \text{(20c)}$$

$$N' = \frac{S_o}{Q^2} \quad \text{(20d)}$$

$$P' = n^2 \quad \text{(20e)}$$

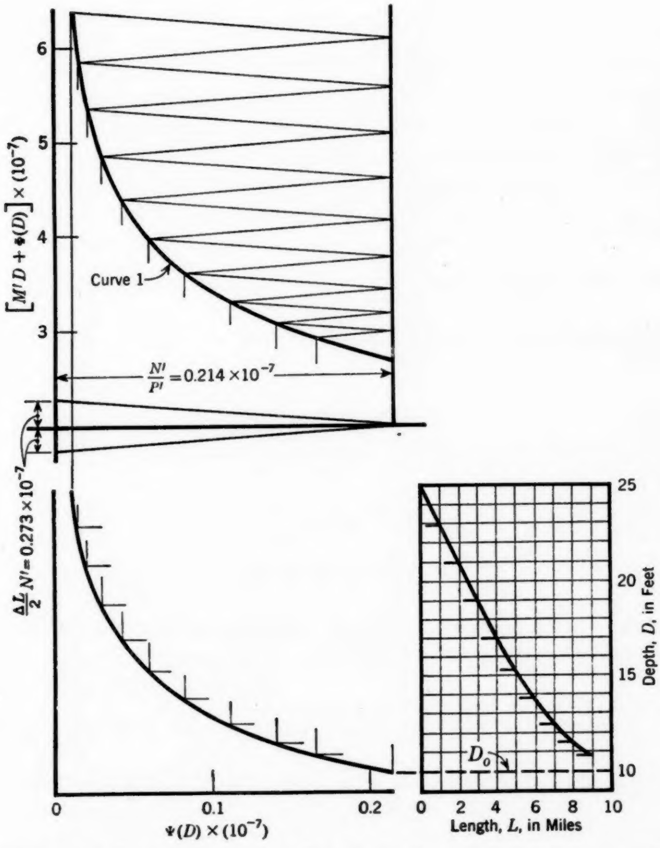


Fig. 8.—Graphical Determination of a Backwater Curve for a Trapezoidal Channel

The graphical solution of Eq. 19 is then the same as that for Eq. 18. Values of $\Phi(D)$ and $\Psi(D)$ can be calculated directly, or the chart of Fig. 7 may be used. Fig. 8 shows the graphical solution of a backwater curve in a trapezoidal channel for which curves 1 and 3 were plotted from the functions obtained from Fig. 7.

The factors for this channel are: $Z = 1$; $b_o = 100$ ft; $Q = 6220$ cu ft per sec; $n = 0.022$; $S_o = 0.0004$; and $D_1 = 25$ ft.

Surge Tanks.—A typical surge tank arrangement is depicted in Fig. 9. The letter symbols used in that figure are as follows: Q is the flow in the conduit; A_c is the conduit area; L is the conduit length; A_t is the tank area; I is the flow to the turbine; H_c is the total head loss in the conduit; H_r is the total head loss in the riser; $H = H_c + H_r$ = the total head loss; and y is the height of the water surface in the tank above the reservoir level.

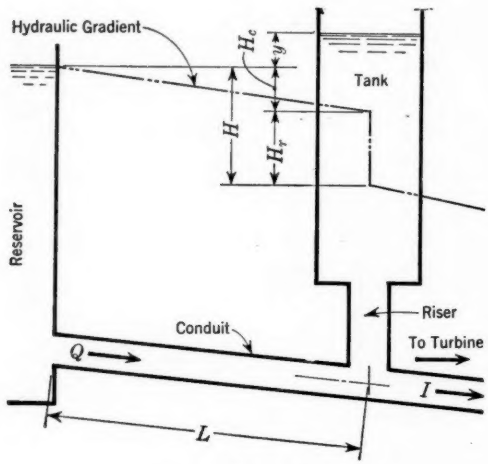


FIG. 9.—SURGE TANK

The following differential equations govern the action of a surge tank;

$$\frac{L}{g\,A_c}\frac{dQ}{dt} = -\,(y + H)\,..(21)$$

and

$$A_t\frac{dy}{dt} = Q - I\,...(22)$$

These equations may be solved graphically, if the following assumptions are made:

$$y_2 = y_1 + \frac{dy}{dt}\,(dt)\,\ldots\ldots(23a)$$

$$y = \tfrac{1}{2}\,(y_1 + y_2)\,\ldots\ldots(23b)$$

$$Q_2 = Q_1 + \frac{dQ}{dt}\,(dt)\,\ldots\ldots(23c)$$

$$Q = \tfrac{1}{2}\,(Q_1 + Q_2)\,\ldots\ldots(23d)$$

Also assume $I = \bar{I}$ = average flow to turbines during the period dt. Eqs. 21 and 22 may then be put into the form

$$\left(\frac{L}{g\,A_c} + \frac{dt^2}{4\,A_t}\right)\frac{dQ}{dt} + H = -\,y_1 - (Q_1 - \bar{I})\,\frac{dt}{2\,A_t}\,\ldots\ldots(24)$$

Furthermore, if

$$\Delta T = dt\,\ldots\ldots(25a)$$

$$M = \frac{1}{\dfrac{L}{g\,A_c} + \dfrac{\Delta T^2}{4\,A_t}}\,\ldots\ldots(25b)$$

and

$$N = \frac{\Delta T}{2\,A_t}\,\ldots\ldots(25c)$$

then

$$\frac{dQ}{M\,dt} + H = -y_1 - N(Q_1 - \bar{I}) \qquad (26)$$

and

$$y_2 = y_1 + N(Q_1 - \bar{I}) + N(Q_2 - \bar{I}) \qquad (27)$$

Eq. 26 is similar in form to Eq. 1 and can be solved in a similar manner.

For the specific problem of instantaneous and complete reject of load by the turbine $\bar{I} = 0$ and Eqs. 26 and 27 become

$$\frac{dQ}{Mdt} + H = -y_1 - N\,Q_1 \qquad (28)$$

and

$$y_2 = y_1 + N(Q_1 + Q_2) \qquad (29)$$

The surge tank in this problem is subject to instantaneous shutdown from an initial flow (Q) equal to 760 cu ft per sec. The other factors pertinent to the problem are: $A_t = 228$ sq ft; $A_c = 78.4$ sq ft; $L = 450$ ft; $H = 31.1 \times 10^{-6}\,Q^2$; and $\Delta T = 2.5$ sec. Using Eq. 25, the value of M is found to be 5.41 and N equals 0.00546. The graphical solution of these equations is shown on Fig. 10(a). Curve 1 is plotted with ordinates of Q and abscissas of H. Line 2 is drawn at slope of 1 on 1 through the intersection of the vertical axis with reservoir level ($y = 0$). Line 3 is drawn at slope $\frac{1}{N}$. Lines 4 and 5 are drawn with slopes $+\frac{M\,\Delta T}{2}$ and $-\frac{M\,\Delta T}{2}$. If y_1 and Q_1 are known, the value of Q_2 can be found as follows:

a. Project line 6 horizontally from Q_1;

b. Project line 7 horizontally from y_1 to line 2, and line 8 vertically to the horizontal axis;

c. Draw line 9 parallel to line 3, and line 10 vertically downward from the intersection with line 6;

d. From the intersection of line 6 with curve 1, draw line 11 parallel to line 4 until reaching line 10;

e. Draw line 12 parallel to line 5 to intersect curve 1; and

f. From the above intersection, draw line 13 horizontally to the end of period ΔT, giving the value of Q_2.

To obtain the value of y_2, the following construction is made:

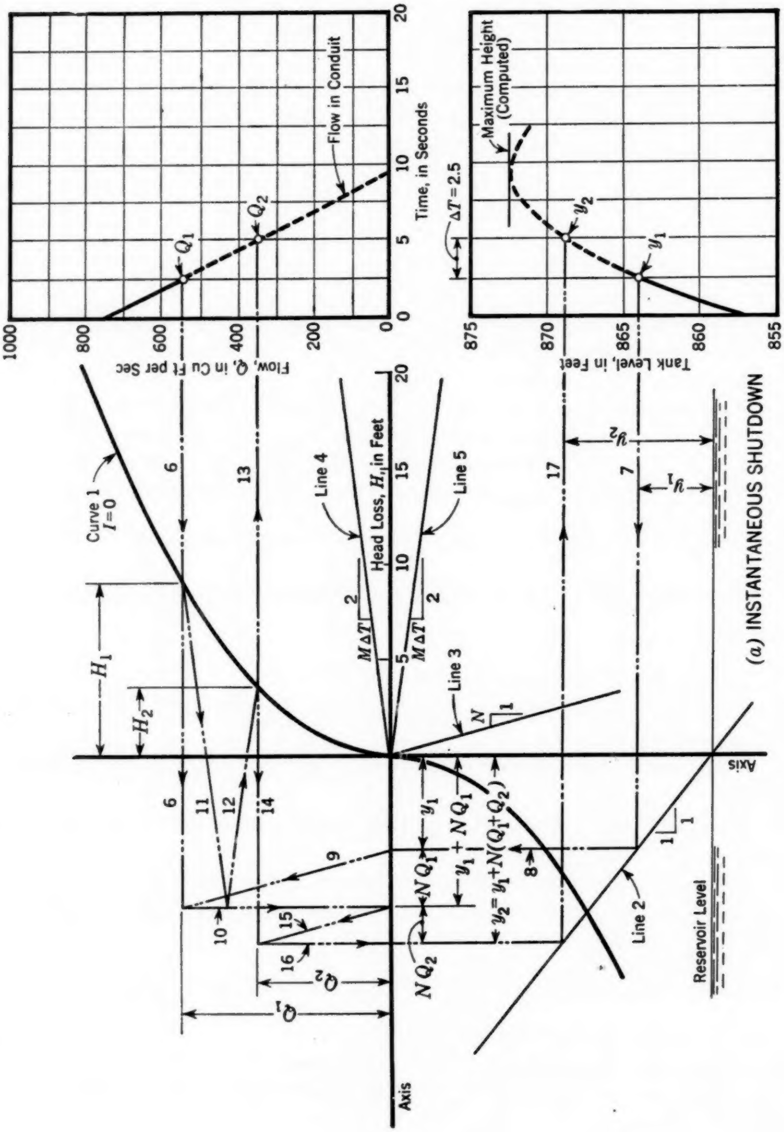
(1) Draw line 14 horizontally from Q_2;

(2) Draw line 15 parallel to line 3 to intersect line 14; and

(3) Draw line 16 vertically to line 2 and draw line 17 horizontally to end of period ΔT, giving y_2.

The foregoing steps are then repeated for the next interval ΔT, using new values of y_1 and Q_1.

In cases of gradual reject or acceptance of load by the turbines Eqs. 26 and 27 must be satisfied. If I is not equal to zero, then H becomes a function of I



(a) INSTANTANEOUS SHUTDOWN

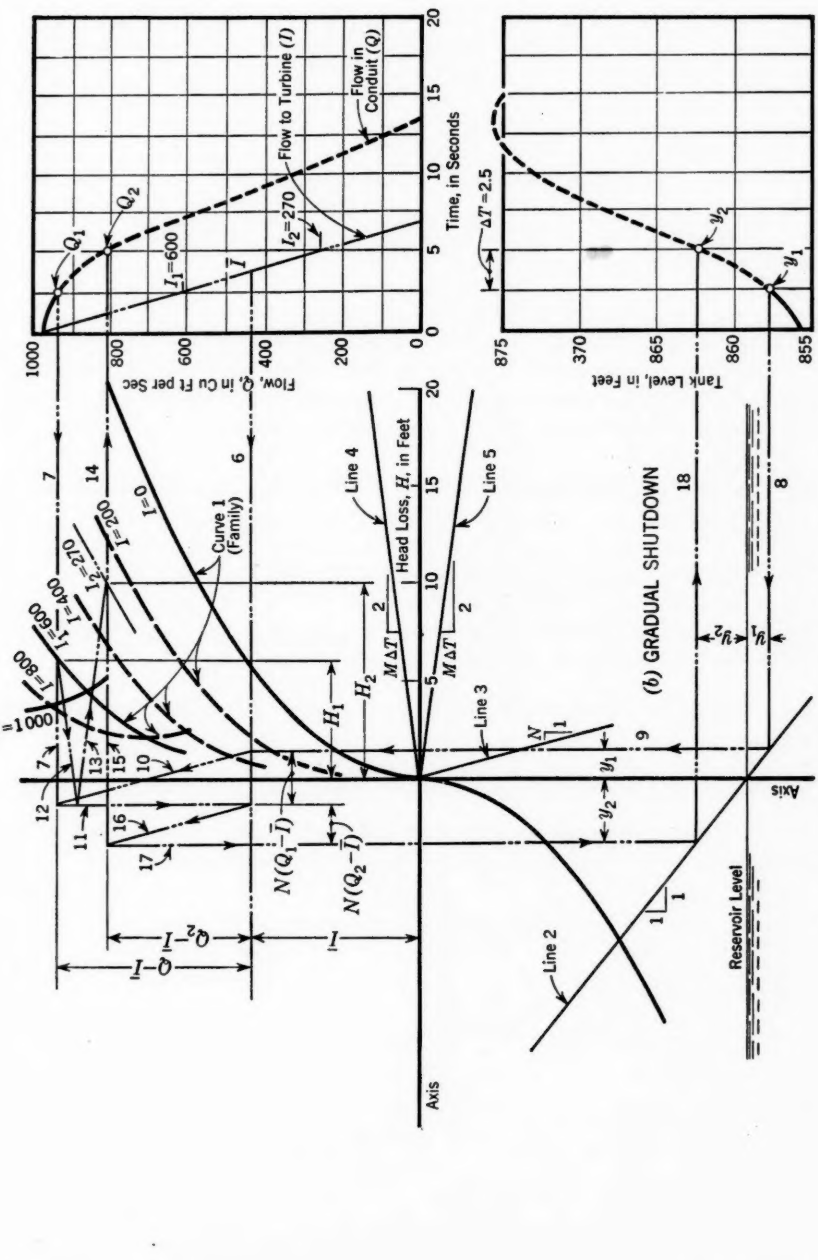


FIG. 10.—GRAPHICAL DETERMINATION OF WATER SURFACE CURVE FOR A SURGE TANK

as well as of Q, and the problem is similar to that of flood routing wherein wedge storage is considered. The solution is as shown on Fig. 10(b). In this case the turbine shuts down gradually, in a period of 7 sec, from an initial flow (Q) of 970 cu ft per sec. The head loss (H) is to conform with Eq. 30 $[3.55\, Q^2 + 27.5\,(Q - I)^2] \times 10^{-6}$ and all other factors are the same as in the case of instantaneous shutdown.

Curve 1 becomes a family of curves, for which I is a parameter as H, in general, may be expressed as

$$H = H_c + H_r = C_1 Q^2 + C_2 (Q - I)^2 \qquad (30)$$

Lines 2, 3, 4, and 5 are constructed as for instantaneous shutdown. The steps taken for the solution are as follows:

(a) Draw line 6 horizontally from $\bar{I}$ for period;
(b) Project line 7 from Q_1 and lines 8 and 9 from y_1;
(c) Draw line 10 parallel to line 3 from the intersection of lines 6 and 9;
(d) Draw line 11 vertically through the intersection of lines 7 and 10;
(e) From the intersection of line 7 with curve 1, corresponding to I_1, draw line 12 parallel to line 4;
(f) From the intersection of lines 12 and 11 draw line 13 parallel to line 5 to the intersection with curve 1 corresponding to I_2;
(g) Draw line 14 horizontally to the right to give Q_2 at end of period ΔT;
(h) Draw line 15 horizontally to the left from Q_2;
(i) Draw line 16 parallel to line 3; and
(j) From the intersection of lines 11 and 16 project line 17 downward to line 2 and line 18 horizontally to give y_2 at the end of period ΔT.

The process is then repeated for the next period ΔT, with new values of y_1 and Q_1.

Extensions to Second Order Differential Equations

The methods of solution of first order differentials may be extended to include solutions of certain second order differential equations such as

$$\frac{d^2y}{dx^2} + A_1 \frac{dy}{dx} + A_2 y = A_3 \qquad (31)$$

With the assumption that

$$y = y_1 + \frac{dy}{dx}\frac{\Delta x}{2} \qquad (32)$$

then

$$\frac{d\left(\frac{dy}{dx}\right)}{dx} + \left(A_1 + A_2 \frac{\Delta x}{2}\right)\frac{dy}{dx} = A_3 - A_2 y_1 \qquad (33)$$

If

$$M = A_1 + A_2 \frac{\Delta x}{2} \qquad (34a)$$

and

$$\psi = \frac{dy}{dx} = \frac{\psi_1 + \psi_2}{2} \qquad \text{(34}b\text{)}$$

then

$$\frac{d\psi}{M\,dx} = \psi + \frac{A_3}{M} - \frac{A_2}{M}y_1 \qquad \text{(35)}$$

and

$$y_2 = y_1 + \frac{\psi_1 + \psi_2}{2}\Delta x \qquad \text{(36)}$$

The solutions of Eqs. 35 and 36 are similar to the preceding examples, with the simplifications that curve 1 is a straight line.

Summary

Most hydraulic problems are of such nature that graphical solutions can give an accuracy well in keeping with the results desired. Trial and error solutions can be avoided as shown herein. The advantages of graphical solutions lie in the continuous visual check against gross errors of individual numerical values, the simple mechanical operations involved once the basic curves are plotted, the facility with which parameters may be varied, and the adaptability to spot checking.